LOAD FLOW

We should be able to analyze the performance of power systems both in normal operating conditions and under fault (short-circuit) condition. The analysis in normal steady-state operation is called power-**flow study** (load-flow study) and it targets on determining the ***voltages, currents, and real and reactive power flows in a system under a given load conditions***.
The purpose of power flow studies is to plan ahead and account for various hypothetical situations.

1. What happens if a transmission line within the power system properly supplying loads must be taken off line for maintenance? Can the remaining lines in the system handle the required loads without exceeding their rated parameters?
2. What happens if a PV or a wind plant is connected to the grid?
3. How can we verify if a market transition is feasible from a technical point of view?

The load flow analysis is useful to ***assess whether a power system is operated in "acceptable" conditions of:***

- Economy (e.g. minimum cost of production)
- Quality (e.g. voltage or current constraints)
- Security (e.g. stability margins)

From a mathematical point of view, the load flow equations are ***nonlinear algebraic equations***; as a consequence there is the need of a CPU source in order to perform such calculations.
In order to write down the load flow equations, one has first to model:

1. the generators and the loads (i.e. the power injections into the network)
2. the network (i.e. the infrastructure)

Network model

In the following, a suitable network model is presented, relating the voltages at the network buses with the currents injected into the network at the same buses. In this model, the infrastructure is supposed to be ***linear***, as will be clarified looking at the final equations. From now on, all the quantities have to be intended as per unit with respect to a suitable base.
Consider a network with N nodes and L branches, applying the Kirchhoff voltage law and the Ohm law, the relations between the branch currents and the node voltages can be written as:

$$[J] = [D][A][V] \tag{1}$$

being V the vector of the node voltages and J the vector of the branch currents, i.e:

$$\begin{aligned} [V] &= [V_1, V_2, ..., V_N]^T \\ [J] &= [J_1, J_2, ..., J_L]^T \end{aligned} \tag{2}$$

The involved matrices are the **incidence nodal matrix** A (size (L,N)) and a diagonal matrix D whose components are the inverse of the branch impedances (size (L,L)).

The incidence matrix A has one column for each node of the grid and one row for each branch and is defined as follows:
If a branch runs from node a to node b, the row corresponding to that branch has 1 in column a and -1 in column b; all other entries in that row are 0.

The vector of the currents $\left[I\right]=\left[I_1,I_2,...,I_N\right]^T$ injected in each node can be evaluated using the Kirchhoff current law, obtaining:

$$\begin{aligned}&[I]=[A]^T[D][A][V]+[D_{shunt}][V]=\\&\left\{[A]^T[D][A]+[D_{shunt}]\right\}[V]\end{aligned} \tag{3}$$

Being D_{shunt} a diagonal matrix whose components are the inverse of the impedances between each node and ground (size (N,N)).

If we introduce the **network admittance matrix** Y defined as:

$$[Y]\triangleq[A]^T[D][A]+[D_{shunt}] \tag{4}$$

then

$$[I]=[Y][V] \tag{5}$$

It can be shown that :

1) The diagonal elements Y_{ii} equal the sum of all the admittances connected to node i.
2) The other elements Y_{ij} are the opposite of the admittances between nodes i and j.

The diagonal elements of Y are called self-admittance or driving-point admittances of the nodes; the off-diagonal elements are called mutual admittances or transfer admittances of the nodes.

From a mathematical point of view, the network admittance matrix is symmetrical and singular if and only if D_{shunt}=0; i.e. if no connection to ground is present.

As said before, equation (5) represents the network model and, from a mathematical point of view, defines a linear relationship between bus voltages and injected currents.

Generators and loads

Both generators and loads will be treated as (p.u.) complex power injections at the node in which they are located, that is to say:

$$\left[\dot{S}\right]_i=[P]_i+j[Q]_i=\dot{V}_i\dot{I}_i^* \tag{6}$$

being P, Q and S the vectors (size (N,1)) of active, reactive and complex power injected into the network at the N buses.

Load flow equations

Now, combining (6) and(5) and writing down the resulting equations component by component, it readily follows:

$$\begin{aligned}&\dot{S}_h=P_h+jQ_h=V_he^{j\delta_h}\sum_{k=1}^{N}\left(Y_{hk}\right)^*V_ke^{-j\delta_k}\\&h=1..N\end{aligned} \tag{7}$$

having indicated with:

$$\dot{V}_h = V_h e^{j\delta_h} \tag{8}$$

the phasor of the voltage at bus h.
If we indicate the real and imaginary part of each entry of the network admittance matrix respectively with G_{hk} and B_{hk}, i.e. if $Y_{hk}=G_{hk}+jB_{hk}$, then (7) becomes:

$$\begin{aligned}
P_h &= V_h \sum_{k=1}^{N} V_k \left[G_{hk} \cos\left(\delta_h - \delta_k\right) + B_{hk} \sin\left(\delta_h - \delta_k\right) \right] \\
Q_h &= V_h \sum_{k=1}^{N} V_k \left[-B_{hk} \cos\left(\delta_h - \delta_k\right) + G_{hk} \sin\left(\delta_h - \delta_k\right) \right] \\
h &= 1..N
\end{aligned} \tag{9}$$

which is the final form of the load flow equations.
From a mathematical point of view, (9) is a nonlinear algebraic system consisting of 2N equations in 4N real unknowns (the voltage amplitude, the voltage phase angle, the injected active power and the injected reactive power for each network bus).

This means that 2N more assigments are necessary in order to solve the problem.
To do this, a physical insight is necessary that allows to perform a sort of classification of the network buses, that can be grouped into three main categories.
1. **Load bus (PQ bus)** – a bus at which the real and reactive power are specified, and for which the bus voltage will be calculated. Real and reactive powers supplied to a power system are defined to be positive, while the powers consumed from the system are defined to be negative. All busses having no generators are load busses.
2. Generator bus (PV bus) – a bus at which the magnitude of the voltage is kept constant by adjusting the field current of a synchronous generator on the bus (increasing the field current of the generator increases both the reactive power supplied by the generator and the terminal voltage of the system). We assume that the field current is adjusted to maintain a constant terminal voltage V_h. We also know that increasing the prime mover's governor set points increases the power P_h that the generator supplies to the power system. Therefore, we can control and specify the magnitude of the bus voltage and real power supplied.
3. Slack bus (swing bus) – a special generator bus serving as the reference bus for the power system. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1e^{j0}$ pu). The real and reactive powers are uncontrolled: the bus supplies whatever real or reactive power is necessary to make the power flows in the system balance.

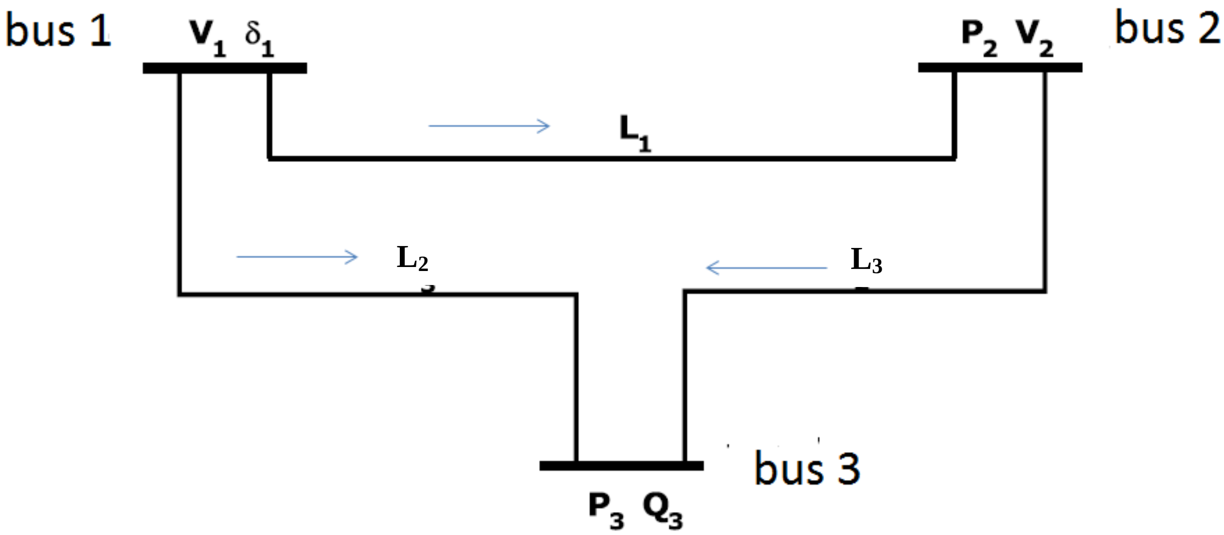
This way, for each node, two more pieces of information are added, thus allowing to preserve the balance between the number of equations and the number of unknowns (2N equations and 2N unknowns).
However, as the reactive power of all the PV buses is unknown, the corresponding equation is not a « real equation » as its free term is unknown. The same applies for both active and reactive power equations at the slack bus.
So consequently, the effective number of equations necessary to to be solved is $2N_{PQ}+N_{PV}$, having indicated with N_{PQ} the number of PQ buses and with N_{PV} the number of PV buses.
After the solution of the $2N_{PQ}+N_{PV}$ nonlinear system, the phasor (8) is known for each bus, which allows to calculate :

- the remaining active and reactive power injections
- the current, active power and the reactive power flows into the infrastructure
- any other desired quantity (i.e. power factor at a specific bus, line loadings etc.)

Example

Let us consider the following three buses HV network, characterized by the following data:

- bus one is the slack (V_1 and δ_1 assigned)
- bus two is PV (P_2 and V_2 assigned)
- bus three is PQ (P_3 and Q_3 assigned)
- the three lines presented negligible resistance and shunt capacitance (i.e. their impedances are purely reactances, indicated in the following with x_1,x_2,x_3)



Nodal incidence matrix :

$$A=\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 0 & 1 & -1\end{bmatrix} \tag{10}$$

Matrix D :

$$[D]=\begin{bmatrix}\frac{1}{jx_1} & 0 & 0\\ 0 & \frac{1}{jx_2} & 0\\ 0 & 0 & \frac{1}{jx_3}\end{bmatrix} \tag{11}$$

Matrix D_{shunt}=0

Matrix Y

$$[Y]=\begin{bmatrix}\frac{1}{jx_1}+\frac{1}{jx_2} & -\frac{1}{jx_1} & -\frac{1}{jx_2}\\ -\frac{1}{jx_1} & \frac{1}{jx_3}+\frac{1}{jx_1} & -\frac{1}{jx_3}\\ -\frac{1}{jx_2} & -\frac{1}{jx_3} & \frac{1}{jx_3}+\frac{1}{jx_2}\end{bmatrix} \tag{12}$$

Power flow equations :

Bus 1 : no equations.

Bus 2 :

$$
\begin{aligned}
P_2 &= V_2 \sum_{k=1}^{3} V_k \left[B_{2k} \sin\left(\delta_2 - \delta_k\right) \right] = \\
&= V_2 V_1 \frac{\sin\left(\delta_2 - \delta_1\right)}{x_1} + V_2 V_3 \frac{\sin\left(\delta_2 - \delta_3\right)}{x_3}
\end{aligned}
\tag{13}
$$

Bus 3 :

$$
\begin{aligned}
Q_3 &= V_3 \sum_{k=1}^{3} V_k \left[-B_{3k} \cos\left(\delta_3 - \delta_k\right) \right] = \\
&= V_3^2 \left(\frac{1}{x_2} + \frac{1}{x_3} \right) - V_3 V_1 \frac{\cos\left(\delta_3 - \delta_1\right)}{x_2} - V_3 V_2 \frac{\cos\left(\delta_3 - \delta_2\right)}{x_3}
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
P_3 &= V_3 \sum_{k=1}^{3} V_k \left[B_{3k} \sin\left(\delta_3 - \delta_k\right) \right] = \\
&= V_3 V_1 \frac{\sin\left(\delta_3 - \delta_1\right)}{x_2} + V_2 V_3 \frac{\sin\left(\delta_3 - \delta_2\right)}{x_3}
\end{aligned}
\tag{15}
$$

Unknowns : V_3,δ_3,δ_2.

The system (13), (14), (15) and can be solved with the aid of a **numerical method**.

A simple case

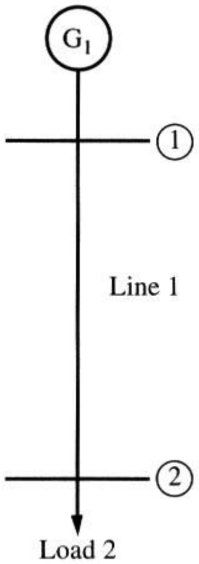


Table of Busses:

Bus 1	Slack bus
Bus 2	Load bus

Line 1 is only a reactance X. Load absorbs P_L and Q_L.

$$
\begin{aligned}
P_2 &= -P_L = \frac{V_1 V_2 \sin\left(\delta_2 - \delta_1\right)}{X} \\
Q_2 &= -Q_L = -\frac{V_1 V_2 \cos\left(\delta_2 - \delta_1\right)}{X} + \frac{V_2^2}{X}
\end{aligned}
\tag{16}
$$

Find V_2 and δ_2

$$\frac{V_2^4}{X^2} - \left[\frac{V_1^2}{X^2} - 2\frac{Q_L}{X}\right]V_2^2 + \left[P_L^2 + Q_L^2\right] = 0 \Rightarrow$$

$$t = V_2^2 \Rightarrow \frac{t^2}{X^2} - \left[\frac{V_1^2}{X^2} - 2\frac{Q_L}{X}\right]t + \left[P_L^2 + Q_L^2\right] = 0 \qquad (17)$$

$$\Delta = \left[\frac{V_1^2}{X^2} - 2\frac{Q_L}{X}\right]^2 - \frac{4}{X^2}\left[P_L^2 + Q_L^2\right]$$

If P_L and Q_L are « too big », no solution exists, which means that the load is requesting a too large amount of power.

Moreover, as t must be positive, the following condition must hold :

$$\left[\frac{V_1^2}{X^2} - 2\frac{Q_L}{X}\right] > 0 \qquad (18)$$

which means that the voltage at the beginning of the line should be sufficiently big.

If such conditions are satisfied, then two possible solutions exist for t, which means that four real solutions exist for the voltage at the load bus. Two of them are negative and so must be disregarded, while two are positive which, from a mathematical point of view, can both be accepted.

For physical reasons, the one closer to V_1 is acceptable.

Numerical example

X= 0.5 pu, V1 = $1.0e^{j0^\circ}$ pu.

Real and reactive powers supplied to the loads from the system at bus 2 are $P_L = 0.3$ pu, $Q_L = 0.2$ pu.

Determine voltages at each bus for the specified load conditions.

Solution :

t =0.757071421427143

t=0.042928578572857

v2 =0.870098512484157

v2=0.207192129611280

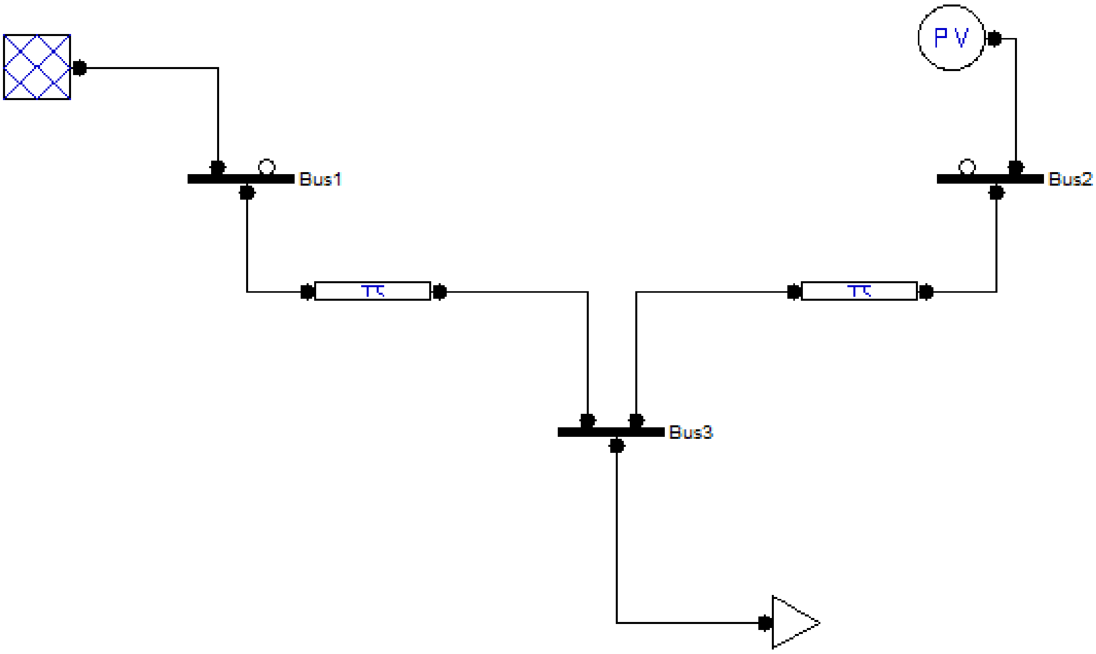
Example with a dedicated software

The next example is solved by means of the dedicated software PSAT.

Consider the following network in which:

- the Load at bus 3 absorbs 100 MW and 60 MVar
- bus two is a PV bus with active power injection of 80 MW and voltage equal to one
- bus one is the slack.

The lines are equal and characterized by a reactance of 0.1 p.u. (400 kV and 100 MVA base) and a R/X ratio of 1/10.



Solving the load flow problem once one has the following report.

POWER FLOW RESULTS

Bus	V [p.u.]	phase [rad]	P gen [p.u.]	Q gen [p.u.]	P load [p.u.]	Q load [p.u.]
Bus1	1	0	0.20923	0.36021	0	0
Bus2	1	0.06181	0.8	0.33019	0	0
Bus3	0.96199	-0.018	0	0	1	0.6

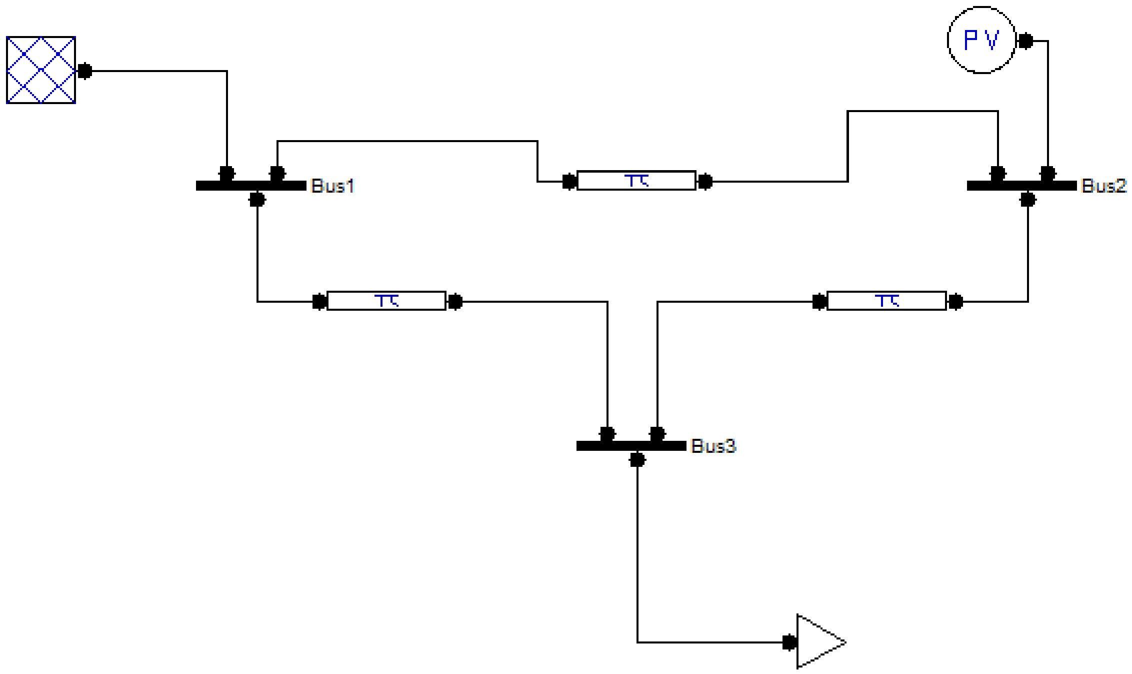
LINE FLOWS

From Bus	To Bus	Line	P Flow [p.u.]	Q Flow [p.u.]	P Loss [p.u.]	Q Loss [p.u.]
Bus1	Bus3	1	0.20923	0.36021	0.00174	0.01643
Bus3	Bus2	2	-0.79251	-0.25622	0.00749	0.07397

LINE FLOWS

From Bus	To Bus	Line	P Flow [p.u.]	Q Flow [p.u.]	P Loss [p.u.]	Q Loss [p.u.]
Bus3	Bus1	1	-0.20749	-0.34378	0.00174	0.01643
Bus2	Bus3	2	0.8	0.33019	0.00749	0.07397

If a line between bus 1 and 2 is added, one has :



with the following results :

POWER FLOW RESULTS

Bus	V [p.u.]	phase [rad]	P gen [p.u.]	Q gen [p.u.]	P load [p.u.]	Q load [p.u.]
Bus1	1	0	0.20794	0.3631	0	0
Bus2	1	0.02016	0.8	0.31337	0	0
Bus3	0.96244	-0.03878	0	0	1	0.6

LINE FLOWS

From Bus	To Bus	Line	P Flow [p.u.]	Q Flow [p.u.]	P Loss [p.u.]	Q Loss [p.u.]
Bus1	Bus2	1	-0.19941	0.02147	0.0004	0.00302
Bus1	Bus3	2	0.40735	0.34162	0.00283	0.02734
Bus3	Bus2	3	-0.59548	-0.28571	0.00471	0.0461

LINE FLOWS

From Bus	To Bus	Line	P Flow [p.u.]	Q Flow [p.u.]	P Loss [p.u.]	Q Loss [p.u.]
Bus2	Bus1	1	0.19981	-0.01845	0.0004	0.00302
Bus3	Bus1	2	-0.40452	-0.31429	0.00283	0.02734
Bus2	Bus3	3	0.60019	0.33182	0.00471	0.0461

Comments:

- The insertion of the line does not affect significantly the voltage amplitude and angles at the three buses
- The insertion of the new line basically affects the active power flows. In the first network, line 2-1 active power is 80 MW (that is to say the over all active power injection from generator at bus two). This means that, if the line is not able to support such active power, the generator cannot produce this power (even if this could be convenient and from an economical point of view). If the line 1-2 is added, then 20 MW flow in it thus shedding the loading of line 2-3 and making the generator at bus two injection feasible.
- The reactive power flow modification is negligible. This is basically due to the fact that we have imposed in that both voltages at bus one and at bus two are 1 p.u. and, as well known, the reactive power flow basically depends on the difference in the magnitude of the voltages at the line ends. This does not apply to the active power flow, which mainly depends on the phase angle difference.