ELECTROMECHANICAL TRANSIENTS

When one talks about **electromechanical transients**, reference is made to all the transients in which one can assume that the electric network can be considered at AC steady-state, which means that it can be modeled with the phasors theory.
This assumption is not able to describe very fast transients, like:

- the effect of direct and indirect lightning overvoltages
- the transients caused by switching operations
- the effects of harmonic currents and voltages

In such cases, one has to adopt the classical equations representing the relationships between instantaneous voltages and currents in all the electric network components. Such transients are called **electromagnetic transients and will be disregarded in these notes.**

1. Introduction

Let us consider a power system working at steady-state. The load flow equations allow to find out:

- the voltages at all the network buses
- the current, active and reactive power injections at all the network buses
- the current and power flows in the network branches

and so to assess the performances of the power system.

Suppose now that a perturbation occurs in a power system working at steady-state (e.g. there is a fault somewhere in the network, a line is disconnected, a large load is connected and so on).

What happens to the power system? How can we describe it?

First of all, let's try to get a physical insight of the possible consequences of a perturbation on a power system.

As well known, each generator present in the network is governed by a motion equation stating that:

$$P_m - P = 2H \frac{d\omega}{dt} \tag{1}$$

being P_m the prime mover power and P the electromagnetic power, that is to say the power which is injected into the electric network. As a consequence, if something changes in the network, also the power injected in the network by the machine will change. In this situation, if the prime mover does not change its power, the machine speed will change.
Now suppose that many machines are present in the considered power system; the speed rate of change basically depends both on the change of the power requested to the specific machine but also on the inertia constant H of the machine itself.
So, two scenarios can arise:

1. the machines experience different accelerations. This way, their speeds become different from each other and, as a consequence, also the machine angles change quite differently. The final result is that the network will experience very high currents and very low voltages, which will cause the protection system tripping. This is the origin of the **blackout**. In this case, one typically states that the machines **loose synchronism**.

2. The machine experience accelerations very close to each other. In this case, the network remains interconnected, but there is a frequency transient due to the fact that the machine speeds change. So, there is the necessity to do something in order to **restore the frequency** to the rated value.

The first phenomenon is quite rapid (in about one second from the perturbation, one is able to judge if the system will loose synchronism or not) and it is typically called "**problem of transient stability**"
The second problem (i.e. the necessity to act on the system frequency) is typically referred as "**active power/frequency regulation**"

2. *Transient Stability Problem: Machine and Network*

In this section, the transient stability problem will be investigated more in details first considering a very simple situation which allows to define the conditions under which the synchronism is maintained or lost and then generalizing the procedure to a more complex power system.
Let us consider the following situation in which a synchronous machine is connected to a network (modelled as an infinite bus: that is to say an ideal voltage source imposing the voltage amplitude V_N and angular frequency ω_e supposed to be constant all along the transient) via a couple of transmission lines whose equivalent parallel reactance is X_p.

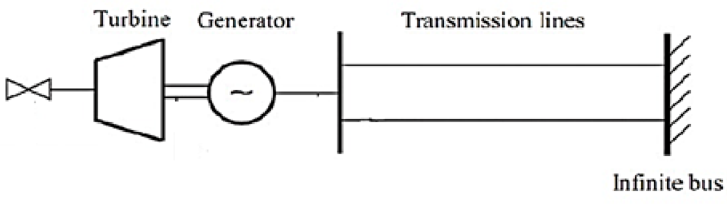


Fig. 1 : Example of network

A simple load flow calculation allows to find out the initial equilibrium point (In the following subscript 0 is used to indicate initial working point values). Assuming that the Network is the slack bus ($V_N e^{j0}$) and the machine is PV bus (P_{e0} and V_{b0} given), one can calculate the phase angle δ_{b0} of the machine voltage and the current flowing in the system, as follows:

$$P_{e0} - \frac{V_N V_{b0}}{X_p} \sin \delta_{b0} = 0 \qquad (2)$$

$$\dot{I}_0 = \frac{\dot{V}_{b0} - V_N}{jX_P} \qquad (3)$$

Modeling the machine as an electromotive force $E'_0 e^{j\delta}$ (amplitude constant and equal to its initial value and phase variable) behind the so-called transient reactance x'_d (see figure) one can then evaluate:

$$E'_0 e^{j\delta_0} = \dot{V}_{b0} + jx'_d \dot{I}_0 \qquad (4)$$

while the prime mover power P_{m0} is P_{e0}

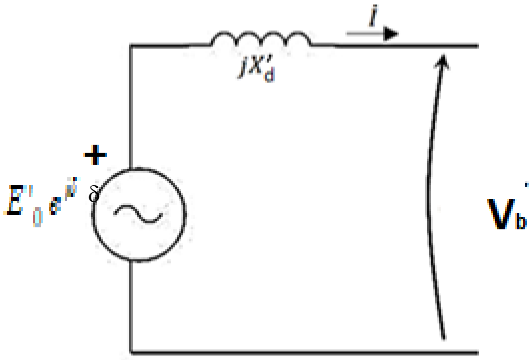


Fig. 2 : Machine equivalent circuit

If one of the two lines is opened, then the reactance becomes $X_{tot}=2X_p+x'_d$, which means that the electromagnetic power decreases and so the machine accelerates according to the following equation.

$$\begin{cases} P_m - P_e = \dfrac{2H}{\omega_N}\ddot{\delta} \\ \delta(0) = \delta_0 \\ \dot{\delta}(0) = 0 \end{cases} \tag{5}$$

Supposing now that during the transient the amplitude of the electromotive force does not change (but the angle does), one has that:

$$P_e = \frac{V_N E_0}{2X_p + x'_d}\sin\delta \tag{6}$$

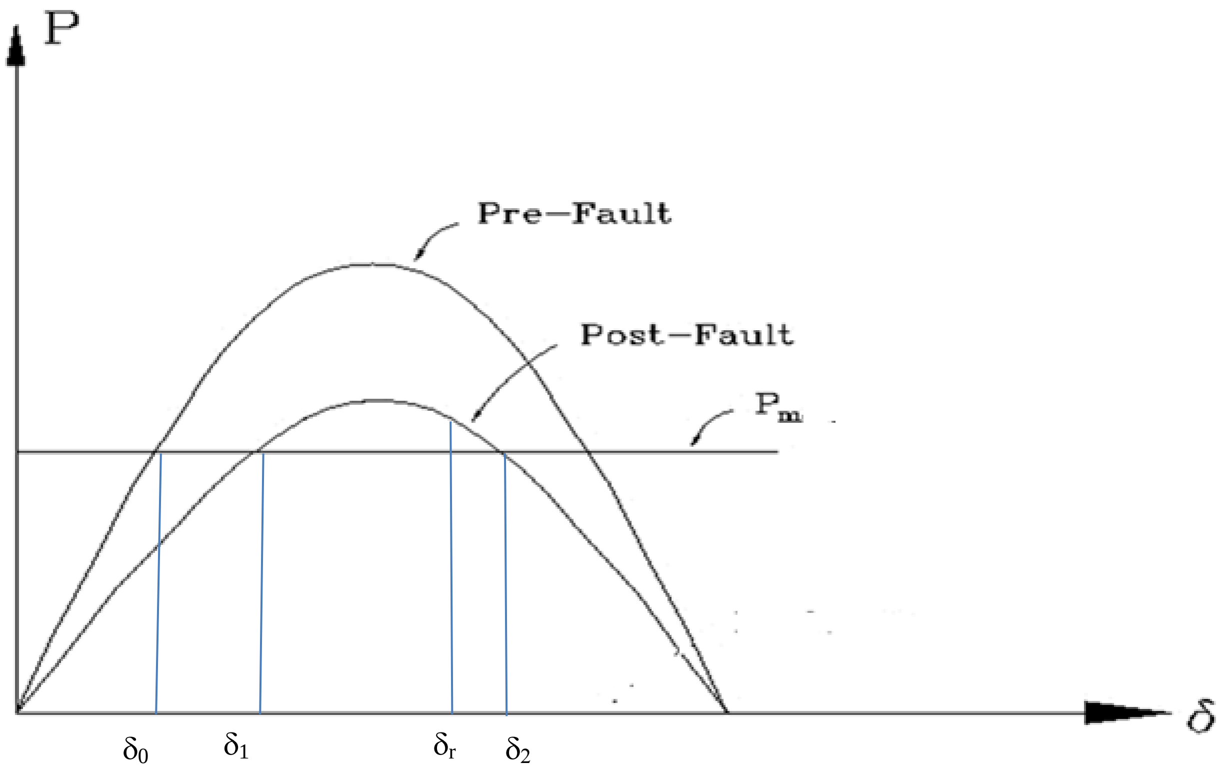


Fig. 3 : P-angle diagram

which can be solved posing:

$$\dot{\delta} = z(\delta) \tag{7}$$

in which z represents the relative speed with respect to the network “rotating at the synchronous speed ω_e”. (remember that $\dot{\delta} = \omega - \omega_e$)

As a consequence, (5) becomes:

$$\begin{cases} P_m - P_e = \dfrac{2H}{\omega_N} z(\delta) z'(\delta) \\ z(\delta_0) = 0 \end{cases} \tag{8}$$

integrating between δ_0 and δ, one has:

$$\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta = \frac{H}{\omega_N} z^2(\delta) \tag{9}$$

So z is given by:

$$z(\delta) = \pm\sqrt{\frac{\omega_N}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta} \tag{10}$$

in order to choose the sign, it is necessary to recall the physical consideration according to which the machine initially accelerates as $P_m(0) > P_e(0)$, so:

$$z(\delta) = \sqrt{\frac{\omega_N}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta} \tag{11}$$

It should be observed that (11) holds until $\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta > 0$.

If there exists δ_r such that $\int_{\delta_0}^{\delta_r}(P_m - P_e)\,d\delta = 0$ and $P_m(\delta_r) < P_e(\delta_r)$, then it is possible to join (11) with the solution of the following problem:

$$\begin{cases} P_m - P_e = \dfrac{2H}{\omega_N} z(\delta) z'(\delta) \\ z(\delta_r) = 0 \end{cases} \tag{12}$$

which is formally similar to (11), with the only difference that the sign before the square root must be negative as now the initial acceleration is negative.
In this case, the system is stable, and the speed does not increase indefinitely and, as a consequence, also the angle always results bounded.
So the conditions for the stability are the following:

$$\begin{cases} \int_{\delta_0}^{\delta_r}(P_m - P_e)\,d\delta = 0 \\ P_m(\delta_r) - P_e(\delta_r) < 0 \end{cases} \tag{13}$$

which means that the accelerating area (i.e. the area in which the mechanical power is greater than the electromagnetic one) is smaller than the decelerating one, as can be seen examining the figure below.
In formulas,

$$\int_{\delta_0}^{\delta_1}(P_m - P_e)\,d\delta \le \int_{\delta_1}^{\delta_2}(P_e - P_m)\,d\delta \tag{14}$$

being δ_1 and δ_2 the points of intersection between the new electromagnetic power curve and the mechanical power one.

Recalling that initially the acceleration is positive, the machine speed relative to ω_e is initially increasing (and so positive) at the least in a neighborhood of $t=0^+$. As a consequence the system is unstable if such relative speed remains positive through the transient. This happens if at least one of the two conditions holds:

1. the relative speed is always increasing, which means that $P_m > P_e$ in all the transient.

2. The relative speed is not always increasing, but its minimum point corresponds to a positive value. This means that:

$$\exists\, \delta_2 \;\; t.c. \begin{cases} P_m(\delta_2) - P_e(\delta_2) = 0 \\ \left.\dfrac{dP_m}{d\delta} - \dfrac{dP_e}{d\delta}\right|_{\delta=\delta_2} > 0 \\ \dot{\delta}(t) > 0 \;\forall t \in [0, t_2] \end{cases} \tag{15}$$

which means that there is a point of intersection between the two curves in which the first derivative of the mechanical power is greater than the first derivative of the electromagnetic one and the relative speed has been positive to that point.

Recalling that :

$$\dot{\delta} = z(\delta) = \frac{\omega_N}{H}\sqrt{\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta} \qquad \delta_0 \le \delta \le \delta_2 \tag{16}$$

it readily follows that:

$$\int_{\delta_0}^{\delta_2}(P_m - P_e)\,d\delta \ge 0 \tag{17}$$

which means that the overall area between the initial point and the last intersection point is positive. So, the instability condition can be reformulated as:

$$\exists\, \delta_2 \;\; t.c. \begin{cases} P_m(\delta_2) - P_e(\delta_2) = 0 \\ \left.\dfrac{dP_m}{d\delta} - \dfrac{dP_e}{d\delta}\right|_{\delta=\delta_2} > 0 \\ \displaystyle\int_{\delta_0}^{\delta_2}(P_m - P_e)\,d\delta \ge 0 \end{cases} \tag{18}$$

So, from a graphical point of view, the system results stable if Area 1 is greater than Area 2; ViceVersa it is unstable. For this reason, the method is called Equal Area Criterion (EAC)

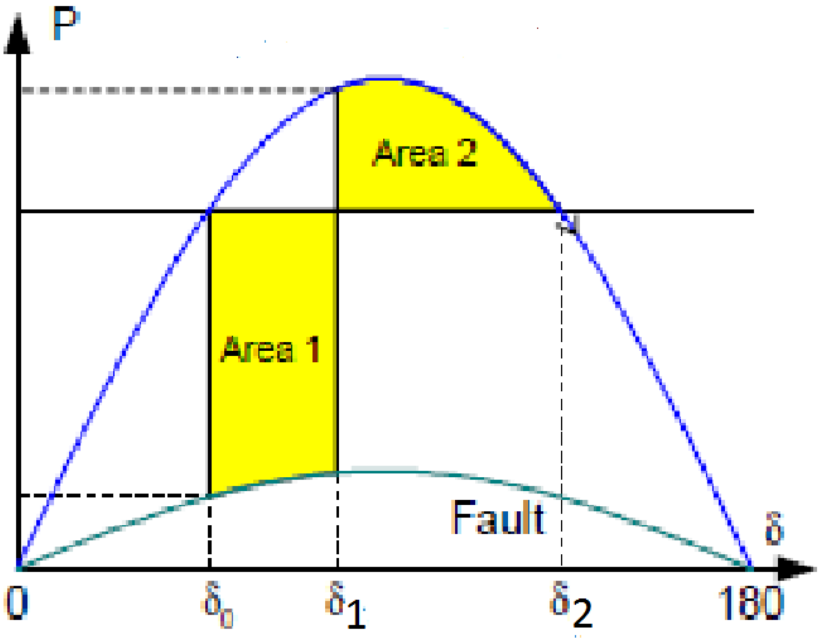


Fig. 4 : Equal Area Criterion

From a quantitative point of view, it is possible to give an indication of "how much the system is stable or unstable" introducing the following stability/instability margins:

$$\eta = -\int_{\delta_0}^{\delta_2} (P_m - P_e)\, d\delta \tag{19}$$

in the stable case, one has that:

$$\eta = -\int_{\delta_0}^{\delta_2} (P_m - P_e)\, d\delta = -\int_{\delta_r}^{\delta_2} (P_m - P_e)\, d\delta > 0 \tag{20}$$

it should be observed that the greater the decelerating area with respect to the accelerating one, the greater is η.

In the unstable case, the margin defined by (19) is negative and, recalling (9), can be expressed by:

$$\eta_u = -\int_{\delta_0}^{\delta_2} (P_m - P_e)\, d\delta = -\frac{H}{\omega_N} z^2(\delta_2) \tag{21}$$

3. *Transient Stability Problem: general case*

Of course, the Equal Area Criterion (EAC) only holds for a structure in which the machine is connected to an infinite bus. How can we assess the network stability in the general case?
To answer this question, it is necessary first to define suitable models for the network and for the machines and then to connect them in order to derive a system of differential equations describing the system dynamics.

I. Network model

Suppose that the network has N_b buses. In that case, the network admittance matrix has N_b rows and N_b columns.
Now, let N_b=N+N_c being N_c the number of load buses and N the machines number.

For the future development, suppose to write down the network admittance matrix as follows:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y_{cc}} & \mathbf{Y_{cm}} \\ \mathbf{Y_{mc}} & \mathbf{Y_{mm}} \end{bmatrix} \tag{22}$$

where:

- Y_{cc} expresses the link between the load currents I_c and the load voltages V_c
- Y_{cm} expresses the link between the load currents I_c and the machine voltages V_m
- Y_{mc} expresses the link between the machine currents I_m and the load voltages V_c.
- Y_{mm} expresses the link between the machine currents I_m and the machine voltages V_m.

Finally, we will express the voltage and current vectors as follows:

$$\mathbf{V} = \left[\mathbf{V}_c^T \quad \mathbf{V}_m^T\right]^T \tag{23}$$

and

$$\mathbf{I} = \left[\mathbf{I}_c^T \quad \mathbf{I}_m^T\right]^T \tag{24}$$

As a consequence, one can rewrite:

$$\begin{cases} [\mathbf{I}_m] = [\mathbf{Y}_{mc}][\mathbf{V}_c] + [\mathbf{Y}_{mm}][\mathbf{V}_m] \\ [\mathbf{I}_c] = [\mathbf{Y}_{cc}][\mathbf{V}_c] + [\mathbf{Y}_{cm}][\mathbf{V}_m] \end{cases} \tag{25}$$

While from the load flow calculations it is suitable to represent the loads by the absorptions of active and reactive power, for the transient stability problems one typically transforms such loads into constant admittances ones. Such transformation can be done using the initial load flow values of complex power and voltage. For any load bus, one can evaluate and the admittance corresponding to the complex power injected by the load and the voltage at the same bus, as follows:

$$y_{ci} = -\frac{P_{li} - jQ_{li}}{V_i^2} \quad i = 1..N_c \tag{26}$$

and define the following matrix:

$$\mathbf{Y}c = diag(y_{ci}) \tag{27}$$

in this way, the loads can be represented by means of linear equations. This suggests the possibility of inserting them into the network admittance matrix, as follows:
From the definition of admittance, it readily follows that for any load bus:

$$[\mathbf{I}_c] = -[\mathbf{Y}_c][\mathbf{V}_c] \quad (28)$$

(the sign - comes out from the fact that the active and reactive power appearing in (26) are supposed to be injected according to the classical load flow formulation).
Then, combining (28) and (25), one has:

$$[\mathbf{V}_c] = -[\mathbf{Y}_c + \mathbf{Y}_{cc}]^{-1}[\mathbf{Y}_{cm}][\mathbf{V}_m] \quad (29)$$

and

$$\begin{aligned}[\mathbf{I}_m] &= -[\mathbf{Y}_{mc}][\mathbf{Y}_c + \mathbf{Y}_{cc}]^{-1}[\mathbf{Y}_{cm}][\mathbf{V}_m] + [\mathbf{Y}_{mm}][\mathbf{V}_m] = \\ &= [\mathbf{Y}_e][\mathbf{V}_m]\end{aligned} \quad (30)$$

having defined

$$\mathbf{Y}_e = \mathbf{Y}_{mm} - \mathbf{Y}_{mc}(\mathbf{Y}_{cc} + \mathbf{Y}_c)^{-1}\mathbf{Y}_{cm} \quad (31)$$

The physical meaning of (30) is the following: the network is now represented by means of a linear relationship between the current injected into the network by the machines and the voltages at the machine buses, as shown in the figure. Definition (31) has the same meaning of the original admittance matrix, with the only difference that now the load buses have been removed and inserted into the network model.

II. Generators model

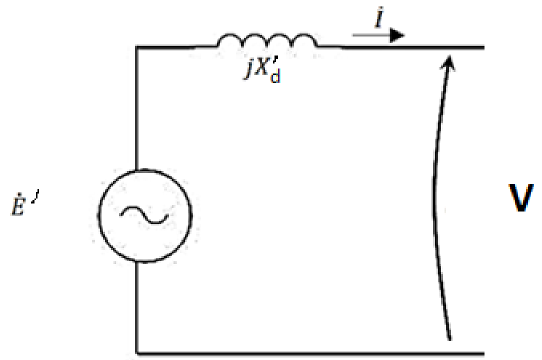


Fig. 5 : Machine equivalent circuit

For the transient stability analysis, each synchronous machine can be modeled according to the equivalent circuit depicted in figure, where the so-called **transient electromotive** force E'_i lays behind a purely reactive impedance jx'_{di} (x'_{di} being the so-called **transient reactance**).
Moreover, the transient electromotive force amplitude is supposed to be constant during the transient ($E'_i = E'_{i0}$), while its phase angle δ_i changes, according to the following equations:

$$\begin{aligned}&\frac{d}{dt}\delta_i = \omega_N[\omega_i - \omega_e] \\ &P_{mi} - P_{ei} = 2H_i\frac{d}{dt}\omega_i \\ &\delta_i(0) = \delta_{0i} \\ &\omega_i(0) = \omega_e \\ &i = 1..N\end{aligned} \quad (32)$$

being ω_i the i-th machine speed, ω_N the rated angular frequency (314 rad/s in Europe), ω_e the network angular frequency and P_{ei} the power injected by the machine into the network that can be calculated as follows:

$$P_{ei}=\mathrm{Re}\left\{\dot{V}_i\dot{I}_i^*\right\}=\mathrm{Re}\left\{\dot{E}_i'\dot{I}_i^*\right\}=\mathrm{Re}\left\{E_{i0}'e^{j\delta i}\dot{I}_i^*\right\} \tag{33}$$

However, the insertion of(33) into (32) is not sufficient to solve the problem as the following quantities are still missing: P_{mi}, δ_{i0}, I_i, E'_{i0}. This opens two problems:

1. the first one is this so-called **initialization** problem that is to say the problem of finding out the initial conditions of the machines dynamic models that are consistent with the initial load flow solution. This is a more general problem which as always to be solved whenever one wants to start a time domain simulation from an initial load flow solution. In this specific case, such problem is solved as follows: after one has performed the load flow calculations, the voltage $V_{bus,i,0}$ at the i-th machine terminals and the active and reactive power P_{i0} and Q_{i0} produced by the machine are at our disposal. As a consequence, the suitable values of the transient electromotive force which guarantees the working point established by the load flow can be calculated by applying the KVL to the circuit in Fig. 5, i.e. :

$$E'_{i0}e^{j\delta_0}=\dot{V}_{bus,i,0}+jx'_{di}\dot{I}_{i,0}=\dot{V}_{bus,i,0}+jx'_{di}\frac{P_{i,0}-jQ_{i,0}}{\dot{V}_{bus,i,0}^*} \tag{34}$$

   Moreover, $P_{mi}=P_{mi0}=P_{i,0}$.

2. The second one consists of combining the dynamic model of the machines with the model of the network and its solution allows finding the current injected by each machine into the network. This can be done combining (32), (33) and (30). If one refers again to Fig. 5, the KVL states that:

$$\begin{aligned}&\dot{V}_{bus,i}+jx'_{di}\dot{I}_i=\dot{E}_i'\\&i=1..N\end{aligned} \tag{35}$$

   Combining now (35) and (30), one has that:

$$\begin{aligned}&\dot{I}_i=\sum_{k=1}^{N}\dot{Y}_{E,ik}\dot{V}_k=\sum_{k=1}^{N}\dot{Y}_{E,ik}\left(\dot{E}'_k-jx'_{dk}\dot{I}_k\right)\Rightarrow\\&\dot{I}_i+\sum_{k=1}^{N}j\dot{Y}_{E,ik}x'_{dk}\dot{I}_k=\sum_{k=1}^{N}\dot{Y}_{E,ik}\dot{E}'_k\Rightarrow\\&\sum_{k=1}^{N}\left(\delta_{ik}+j\dot{Y}_{E,ik}x'_{dk}\right)\dot{I}_k=\sum_{k=1}^{N}\dot{Y}_{E,ik}\dot{E}'_k\end{aligned} \tag{36}$$

   δ_{ik} being the Kronecker operator. So, defining:

$$M_{ik}=\left(\delta_{ik}+j\dot{Y}_{E,ik}x'_{dk}\right) \tag{37}$$

   one has that:

$$\left[\mathbf{I}_m\right]=\left[\mathbf{Y}_{el}\right]\left[\dot{\mathbf{E}}'\right] \tag{38}$$

   where:

$$\left[\mathbf{Y}_{el}\right]=\left[\mathbf{M}\right]^{-1}\left[\mathbf{Y}_e\right] \tag{39}$$

   which can be interpreted, from a physical point of view, as the relationship between electromotive forces and injected currents of the network depicted in Fig. 6.

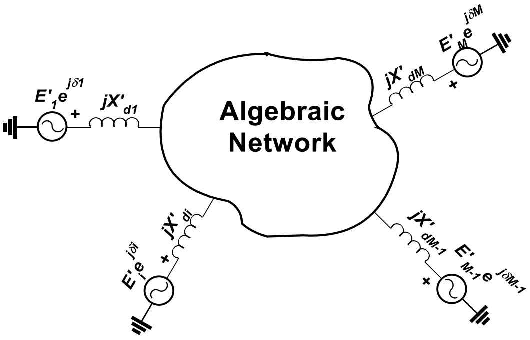


Fig. 6 : Final network model

Now, **Error! Reference source not found.** represents the required link between the currents and the transient electromotive forces to be inserted into (33) for the calculation of the electromagnetic power. Supposing that the generic entry of the new admittance matrix can be expressed as:

$$Y_{el,hk} = G_{el,hk} + jB_{el,hk} \tag{40}$$

Then the electromagnetic power of the ith machine can be expressed as:

$$\begin{aligned} P_{ei} &= \operatorname{Re}\left\{E'_i e^{j\delta_i} \dot{I}_i^*\right\} = \operatorname{Re}\left\{E'_i e^{j(\delta_i)} \sum_{k=1}^{N} Y_{el,ik} E'_k e^{-j(\delta_k)}\right\} = \\ &= \operatorname{Re}\left\{E'_i e^{j(\delta_i)} \sum_{k=1}^{N} \left(G_{el,ik} - jB_{el,ik}\right) E'_k e^{-j(\delta_k)}\right\} = \\ &= E'_i \sum_{k=1}^{N} \left[G_{el,ik} E'_k \cos\left(\delta_i - \delta_k\right) + B_{el,ik} E'_k \sin\left(\delta_i - \delta_k\right)\right] \\ i &= 1..N \end{aligned} \tag{41}$$

which allows to find out the final form of the differential equation system that regulates the electromechanical transient of and network with N machines.

$$\begin{aligned} &\frac{d}{dt}\delta_i = \omega_N \left[\omega_i - \omega_e\right] \\ &P_{mi} - E'_i \sum_{k=1}^{N} \left[G_{el,ik} E'_k \cos\left(\delta_i - \delta_k\right) + B_{el,ik} E'_k \sin\left(\delta_i - \delta_k\right)\right] = 2H_i \frac{d}{dt}\omega_i \\ &\delta_i(0) = \delta_{0i} \\ &\omega_i(0) = \omega_e \\ &i = 1..N \end{aligned} \tag{42}$$

Examining (42), it is apparent that:

- if there are no changes in $Y_{el,hk} = G_{el,hk} + jB_{el,hk}$, then the system is always at steady-state
- whenever something changes in the network, that will be a change in $Y_{el,hk} = G_{el,hk} + jB_{el,hk}$, which will initiate the system dynamics.

From a physical point of view, if neither the prime mover power nor the network structure change, then the steady-state working point established by the loadflow calculation will not change and so the time derivatives of the quantities appearing in (42) will always be zero. The perturbation typically consists of some changes in the network fed by the machine; in this case, even if the machine conditions do not change, the current injected by the machine will change and so will stimulate the system dynamics according to the second of (42).

4. Application

The modern electricity infrastructure represents a complex system composed by generating units, transmission and distribution networks, loads and power electronic devices which is currently operated in accordance to specific economical rules and models. In a modern vision, this network can be represented as a large and complex dynamic system whose operation has to be reliable, secure, and economical [1]. Reliability and security aspects are becoming more and more important due to the increasing penetration of distributed and renewable generating units that in the last decades have significantly changed the operating asset of the electricity system introducing stochastic and sudden power variations and bidirectional flows among transmission and distribution networks. In this context, maintaining transient stability is a fundamental requirement of interconnected power system operation, as it concerns the ability of the system to withstand severe disturbances while ensuring continuity of service. As a consequence, many different power system researchers have focused their attention on solving the two basic problems of:

(i) **defining efficient methods to assess the stability** of a power system subject to a particular contingency and

(ii) **implementing operative actions and controls** in order to achieve this kind of stability.

Due to the complexity of the derived equations, in a real network, evaluating the power system stability can only be done by means of **computer simulations** that numerically integrate (42) (or more complex versions whenever the machine models are more complicated order some other devices like governors or Automatic Voltage Regulators are present). The main appeal of this approach lays in its ability to handle any power system modeling and to provide detailed time domain descriptions of the physical phenomena, while the main weakness is the lack of sensitivity and control information.

Also, time-domain methods remain computationally demanding, despite the progress of computer performance in recent years. To alleviate the above weaknesses, **Lyapunov like direct methods** started being developed in the early sixties [2]–[4]. More recently, Lyapunov-like hybrid methods have been adopted; among them, it is worth mentioning the **SIngle Machine Equivalent (SIME**), which is a hybrid time domain/direct method [5], [6]. In short, it relies on time domain simulations with early termination and calculation of stability limits dictated by the equal-area criterion.

As long as the second problem, is concerned, there are basically two categories of methods for maintaining the stability of a power system. The first one is the so-called **preventive control** and consists of adjusting the system operating point (mainly active power settings), in order to make it able to withstand a set of probable contingencies without losing stability. In the recent years, different algorithms have been developed in order to insert into an Optimal Power Flow problem specific constraints accounting for stability issues (see [7]–[9] for instance). The second group of methods is the so-called **emergency control**, as it operates in real-time and designs an action (typically fast generator tripping and load shedding) in order not to lose the stability of a power system after the occurrence of a contingency.

Stability assessment: The SIME method

The SIME method is based on the observation that all the unstable cases are determined by the irrevocable separation of the machines into two groups: the ones that are accelerating (critical machines) with respect to the center of inertia (COI) of the system and the ones that are decelerating (not critical machines).

For example, let us consider the following network and suppose that a short circuit happens at bus seven, cleared after 0.3 s.

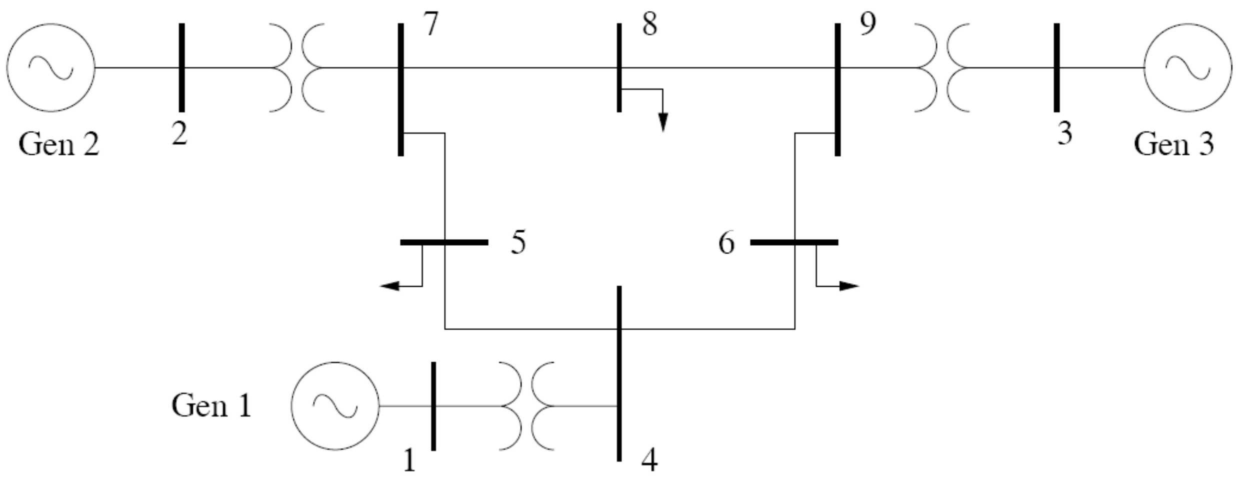


Fig. 7 : Test network

The machine angles, depicted in Fig. 8, show that machines two and three belong to the critical machines set (C), while the not critical set (N) is only composed by machine one.

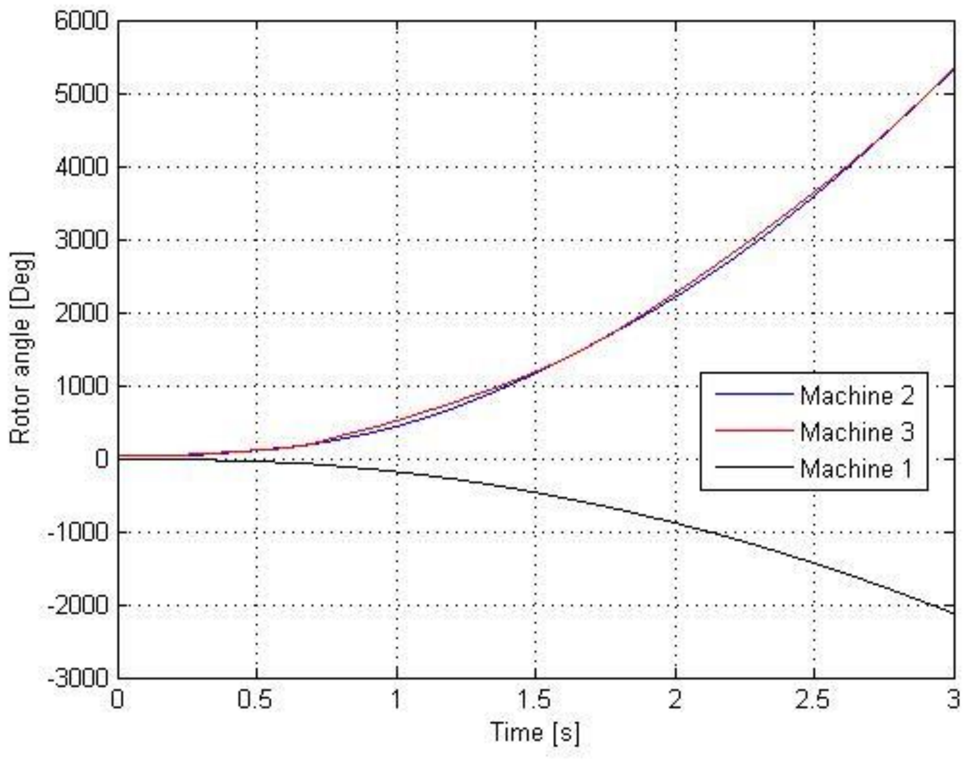


Fig. 8 : machine angles

So, if one performs an unstable simulation, it is easy to rank the machines and divide them into the two clusters, just by sorting them in an increasing order of the machine angles. If a certain time t_{obs}, the difference between two consecutive machines angles is greater than a specified threshold, the simulation is stopped and the two clusters are formed.

At this point, the basic idea is to construct two machines that represent the two clusters, as follows:

$$\begin{cases} \delta_C(t) = M_c^{-1} \sum_{k \in C} M_k \delta_k(t) \\ M_c = \sum_{k \in C} M_k \end{cases} \quad (43)$$

and

$$\begin{cases} \delta_N(t) = M_N^{-1} \sum_{k \in N} M_k \delta_k(t) \\ M_N = \sum_{k \in N} M_k \end{cases} \tag{44}$$

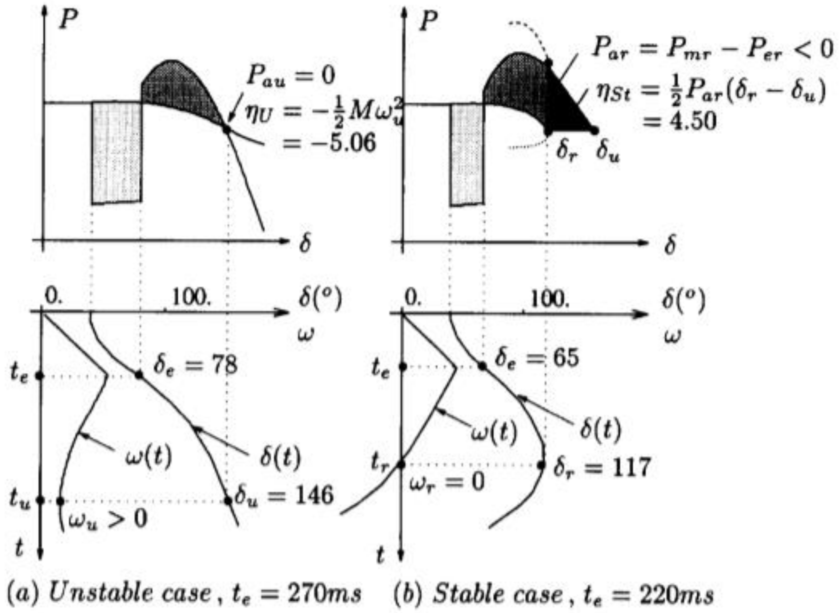
and an equivalent OMIB (One machine Infinite Bus) whose angle is:

$$\delta(t) = \delta_C(t) - \delta_N(t) \tag{45}$$

The powers that determine the dynamics of such equivalent system are defined as:

$$\begin{cases} P_m(t) = M\left(M_C^{-1} \sum_{k \in C} P_{mk}(t) - M_N^{-1} \sum_{k \in N} P_{mk}(t) \right) \\ P_e(t) = M\left(M_C^{-1} \sum_{k \in C} P_{ek}(t) - M_N^{-1} \sum_{k \in N} P_{ek}(t) \right) \\ M = \dfrac{M_N M_C}{M_N + M_C} \end{cases} \tag{46}$$

System (45)-(46) is of the kind (5) and so it is possible to apply the EAC and evaluating the corresponding instability margin.
It should be observed that the EAC can be used to no matter the dynamic model proposed for the machines. The difference with the classic second-order model with infinite bus consists of the fact that in the general case the mechanical power are not constant and the link between the electric power and the network voltages/currents is not (41) anymore. The value of the two quantities are only known in correspondence of the numerical simulation time samples.
Now, it is necessary to perform another simulation (typically with a less severe contingency) which can lead either to a stable or to an unstable situation and to evaluate the corresponding stability/instability margin.
At this point it should be observed that there is a fundamental difference in the evaluation of the two margins. The instability margin can be evaluated easily, because it is sufficient to have at our disposal the OMIB speed in correspondence of the points in which mechanic and electric powers are equal. The operation which is required is typically an interpolation, as it is improbable that the two powers are equal in correspondence of one of the time domain simulation time samples.
The stability margin is, on the other hand, an integral which can be evaluated only by means of trapezoidal rule, as the integrand function is only known in correspondence of the simulation time samples. A more simplified rule is proposed in [5] and [9], that approximate the integral with a triangle constructed on the two integral limits ($\eta = \int_{\delta_r}^{\delta_2} (P_e - P_m)\, d\delta \cong \frac{1}{2}(\delta_u - \delta_r)\left[P_e(\delta_r) - P_m(\delta_r)\right]$)



Then, it is possible to estimate the boundary between the stable and unstable situation just by interpolating or extrapolating the two obtained margins (the interpolation is required if one stable simulation and one unstable simulation has been done, while the extrapolation has to be done in the case of two unstable simulations, see Fig. 9).

This way, for a particular contingency, it is possible to have an estimate of the level of severity that the system can withstand before producing instability, limiting the number of time domain simulations and the consequent CPU effort. The whole process is repeated for a set of credible contingency in order to assess the system security.

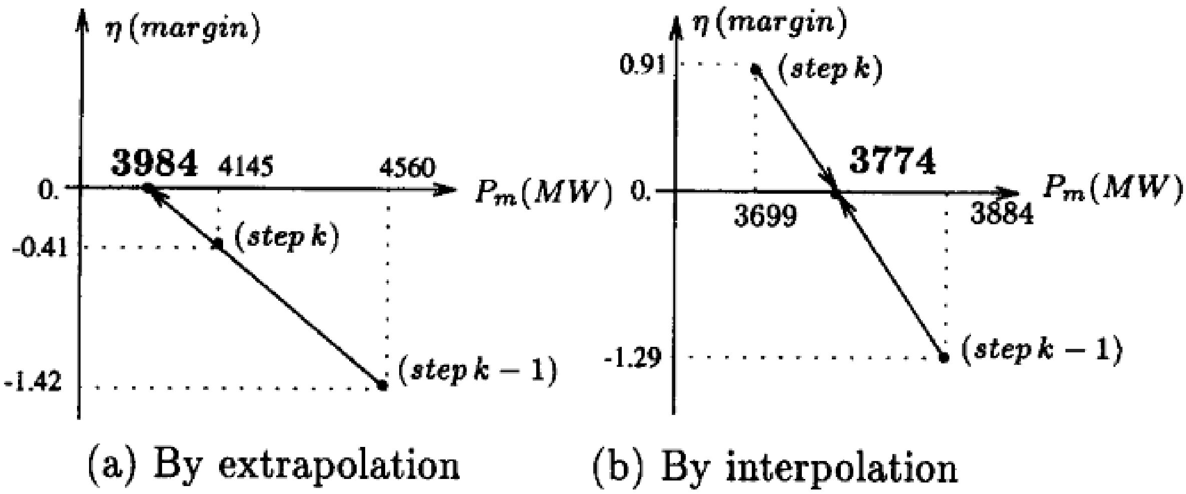


Fig. 9 : interpolation/extrapolation of the stability margins

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